3.2 ⋆ Let's choose our x axis to point due north and y vertically up, and let $\mathbf{v}$ be the velocity of the second fragment just after the explosion. Then conservation of momentum implies that $\frac{1}{2}mv_x = mv_o$ and $\frac{1}{2}mv_y = -\frac{1}{2}mv_o$. Therefore, $\mathbf{v} = (2v_o, -v_o, 0)$ and the requested velocity is $\sqrt{5}v_o$ north at an angle $\theta = \arctan(1/2) = 26.6°$ below the horizontal.

3.4 ⋆⋆ **(a)** Let v be the speed of recoil of the flatcar, so that $u - v$ is the speed of either hobo relative to the ground just after they jump. Conservation of momentum implies that $2m_h(u - v) = m_{fc}v$, from which we find

$$v = \frac{2m_h}{2m_h + m_{fc}}u. \tag{i}$$

(b) Let v' be the recoil speed of the flatcar after the first hobo jumps and v'' that after the second. Conservation of momentum in the first jump works just as in part (a) (except that only one hobo jumps) and we conclude that

$$v' = \frac{m_h}{2m_h + m_{fc}}u\,. \tag{ii}$$

The second jump is more complicated because the flatcar is already moving with speed v'. In this case, conservation of momentum implies that

$$m_h(u - v'') - m_{fc}v'' = -(m_h + m_{fc})v'$$

or

$$v'' = \frac{m_hu + (m_h + m_{fc})v'}{m_h + m_{fc}} = \frac{3m_h + 2m_{fc}}{2m_h + 2m_{fc}}\frac{2m_h}{2m_h + m_{fc}}u = \frac{3m_h + 2m_{fc}}{2m_h + 2m_{fc}}v$$

where the second equality results from substitution of Eq.(ii) (plus some algebra) and the last one from use of Eq.(i). Clearly $v'' > v$, so the second procedure gives the larger final recoil velocity.

3.8 ⋆ **(a)** The condition for the rocket to hover is that $-\dot{m}v_{ex} = mg$. This requires that $-dm/m = g\,dt/v_{ex}$, which integrates to give $-\ln(m/m_o) = gt/v_{ex}$. The maximum time occurs when $m = (1 - \lambda)m_o$, so $t_{max} = -\ln(1 - \lambda)v_{ex}/g$.

(b) If $\lambda = 0.1$ and $v_{ex} = 3000$ m/s, this gives $t_{max} = 32$ seconds.

3.12 ⋆⋆ **(a)** If it uses all the fuel in a single burn, then according to (3.8) the final speed is

$$v = v_{\rm ex} \ln\left(\frac{m_{\rm o}}{0.4m_{\rm o}}\right) = v_{\rm ex}\ln(2.5) = 0.92v_{\rm ex}.$$

(b) After the first stage the speed is

$$v_1 = v_{\rm ex} \ln\left(\frac{m_{\rm o}}{0.7m_{\rm o}}\right) = v_{\rm ex}\ln\left(\frac{1}{0.7}\right)$$

and after the second stage it is

$$v_2 = v_{\rm ex} \ln\left(\frac{0.6m_{\rm o}}{0.3m_{\rm o}}\right) + v_1 = v_{\rm ex}\ln\left(\frac{0.6}{0.3}\times\frac{1}{0.7}\right) = v_{\rm ex}\ln(2.86) = 1.05v_{\rm ex}.$$

3.14 ⋆⋆ The equation of motion (3.29) reads $m\dot{v} = kv_{\rm ex} - bv$, or $m\,dv/(kv_{\rm ex} - bv) = dt$. Since $dm/dt = -k$ we can replace dt by $-dm/k$, and the equation of motion becomes $k\,dv/(kv_{\rm ex} - bv) = -dm/m$. This integrates to give

$$\frac{k}{b}\ln\left(\frac{kv_{\rm ex} - bv}{kv_{\rm ex}}\right) = \ln\left(\frac{m}{m_{\rm o}}\right) \quad \text{or} \quad v = \frac{kv_{\rm ex}}{b}\left[1 - \left(\frac{m}{m_{\rm o}}\right)^{b/k}\right].$$

3.22 ⋆⋆ Let the hemisphere's mass be M and its density be $\varrho = M/V$, where $V = 2\pi R^3/3$ is its volume. The CM position is $\mathbf{R} = \int \varrho\mathbf{r}\,dV/M = \int \mathbf{r}\,dV/V$ where the integral runs over the volume of the hemisphere. By symmetry $X = Y = 0$, while

$$Z = \frac{1}{V}\int z\,dV = \frac{3}{2\pi R^3}\int_0^R r^2 dr \int_0^{\pi/2} \sin\theta\,d\theta \int_0^{2\pi} d\phi\, r\cos\theta = \frac{3}{2\pi R^3}\cdot\frac{R^4}{4}\cdot\frac{1}{2}\cdot 2\pi = \frac{3}{8}R$$

3.25 ⋆ The net force on the particle is just the tension of the string, which is necessarily directed toward the hole in the table at O. Therefore the angular momentum $\boldsymbol{\ell}$ about O is constant. When the particle is travelling in a circle of radius r, the vertical component of $\boldsymbol{\ell} = \mathbf{r}\times\mathbf{p}$ is $\ell_z = rp = rmv = rm(r\omega) = mr^2\omega$. Therefore, the quantity $r^2\omega$ is constant and $r^2\omega = r_{\rm o}^2\omega_{\rm o}$; whence $\omega = (r_{\rm o}/r)^2\omega_{\rm o}$.

3.30 ⋆⋆ **(a)** If a particle is a distance ρ from the axis of rotation and the body turns through an angle $d\phi$, then the particle moves a distance $\rho d\phi$ in the tangential (ϕ) direction. Dividing by dt we conclude that the particle's speed is $v = \rho d\phi/dt = \rho\omega$ in the ϕ direction. That is, $\mathbf{v} = \rho\omega\hat{\boldsymbol{\phi}}$.

(b) The particle's position is $\mathbf{r} = \rho\hat{\boldsymbol{\rho}} + z\hat{\mathbf{z}}$, so its angular momentum is $\boldsymbol{\ell} = \mathbf{r}\times\mathbf{p} = (\rho\hat{\boldsymbol{\rho}} + z\hat{\mathbf{z}}) \times m\rho\omega\hat{\boldsymbol{\phi}} = m\rho^2\omega\hat{\mathbf{z}} - mz\rho\omega\hat{\boldsymbol{\rho}}$. Therefore its z component is $\ell_z = m\rho^2\omega$.

(c) The total angular momentum has

$$L_z = \sum_{\alpha=1}^{N} \ell_{\alpha z} = \sum_{\alpha=1}^{N} m_\alpha \rho_\alpha^2 \omega = I\omega \quad \text{where} \quad I = \sum_{\alpha=1}^{N} m_\alpha \rho_\alpha^2.$$

3.32 ⋆⋆ The sum (3.31) becomes the integral $I = \int \varrho\, dV \rho^2$, where $\varrho = M/V = 3M/(4\pi R^3)$ is the density and $\rho = r\sin\theta$ is the distance of a point from the z axis. Therefore

$$I = \frac{3M}{4\pi R^3}\int_0^R r^4 dr \int_0^\pi \sin^3\theta\, d\theta \int_0^{2\pi} d\phi = \frac{3M}{4\pi R^3}\cdot\frac{R^5}{5}\cdot\frac{4}{3}\cdot 2\pi = \frac{2}{5}MR^2.$$

3.34 ⋆⋆ The CM moves just like a point mass M, so its height is $Y = v_o t - \frac{1}{2}gt^2$, and the time to return to $Y = 0$ is $t = 2v_o/g$. Since there is no torque about the CM, the angular momentum $L = I\omega$ is constant. Therefore $\omega = \omega_o$ is also constant and the number of complete revolutions in the time t is $n = \omega_o t/2\pi = \omega_o v_o/\pi g$. Therefore, he must arrange that $v_o = n\pi g/\omega_o$ where n is an integer.